

EXERCISE – III

SUBJECTIVE QUESTIONS

1. Find the points on the curve $y = x^3$ at which at slope of the tangent is equal to the y -coordinates of the point.

2. Find the equation of tangent to the curve $y = 1 + e^{-2x}$ where it cuts the line $y = 2$.

3. Find equation of tangent and normal to the curve $x^3 + y^3 = 6xy$ at point $(3, 3)$.

4. Find the equation of normal to the curve $x^3 + y^3 = 8xy$ at point where it is meet by the curve $y^2 = 4x$, other than origin.

5. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is

(a) parallel to the line $2x - y + 9 = 0$

(b) perpendicular to the line $2y - x + 1 = 0$

6. Show that for any point of the curve $x^2 - y^2 = a^2$ the segment of the normal from the point to the point of intersection of the normal with the x -axis is equal to the distance of the point from the origin.

7. Prove that the length of segment of all tangents to curve $x^{2/3} + y^{2/3} = a^{2/3}$ intercepted between coordinate axes is same.

8. A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y coordinate is changing 8 times as fast as the x co-ordinate.

9. For the curve $y = 4x^3 - 2x^5$, find all the points at which the tangent passes through the origin.

10. If the tangent at $(1, 1)$ on $y^2 = x(2 - x)^2$ meets the curve again at P , then find coordinates of P .

11. Find the angle of intersection of the curve $y = 2 \sin^2 x$ and $y = \cos 2x$.

12. Find the shortest distance between curves $xy = 9$ and $x^2 + y^2 = 1$

13. Find the point on hyperbola $3x^2 - 4y^2 = 72$ which is nearest to the straight line $3x + 2y + 1 = 0$

14. Show that subnormal at any point on the curve $x^2 y^2 = a^2(x^2 - a^2)$ varies inversely as the cube of its abscissa.

15. The length x of rectangle is decreasing at a rate of 3 cm/min and the width y is increasing at the rate of 2 cm/min, when $x = 10$ cm and $y = 6$ cm, find the rates of changes of

(i) the perimeter, and (ii) the area of the rectangle.

16. If x and y are sides of two squares such that $y = x - x^2$. Find the rate of change of area of second square with respect to the first square when $x = 1$ cm.

17. The tangent to the graph of the function $y = f(x)$ at the point with abscissa $x = a$ forms with the x -axis an angle of $\pi/3$ and at the point with abscissa $x = b$ at an angle of $\pi/4$, then find the value of the integral,

$$\int_a^b f'(x) \cdot f''(x) dx \text{ [assume } f''(x) \text{ to be continuous]}$$

18. A light shines from the top of a pole 50 ft high. A ball is dropped from the same height from a point 30 ft away from the light. How fast is the shadow of the ball moving along the ground $1/2$ sec. later? [Assume the ball falls a distance $s = 16t^2$ ft in ' t ' sec.]

19. Find the abscissa of the point on the curve, $xy = (c - x)^2$ the normal at which cuts off numerically equal intercepts from the axes of co-ordinates.

20. Find the set of values of p for which the equation $|\ln x| - px = 0$ possess three distinct roots is.

21. Find the minimum value of

$$(x_1 - x_2)^2 + \left(\sqrt{2 - x_1^2} - \frac{9}{x_2} \right)^2 \text{ where } x_1 \in (0, \sqrt{2}) \text{ and } x_2 \in \mathbb{R}^+.$$

22. Find the possible values of a such that the inequality $3 - x^2 > |x - a|$ has atleast one negative solution.

23. If the relation between subnormal SN and subtangent ST at any point S on the curve $by^2 = (x + a)^3$

is $p(SN) = q(ST)^2$, then find value of $\frac{p}{q}$ in terms of b and a .

24. In the curve $x = a \left(\cos t + \log \tan \frac{t}{2} \right)$, $y = a \sin t$, show that the portion of the tangent between the point of contact and the x -axis is of constant length.